Physikalisches Fortgeschrittenenpraktikum

Landéfaktor des Myons

– report –

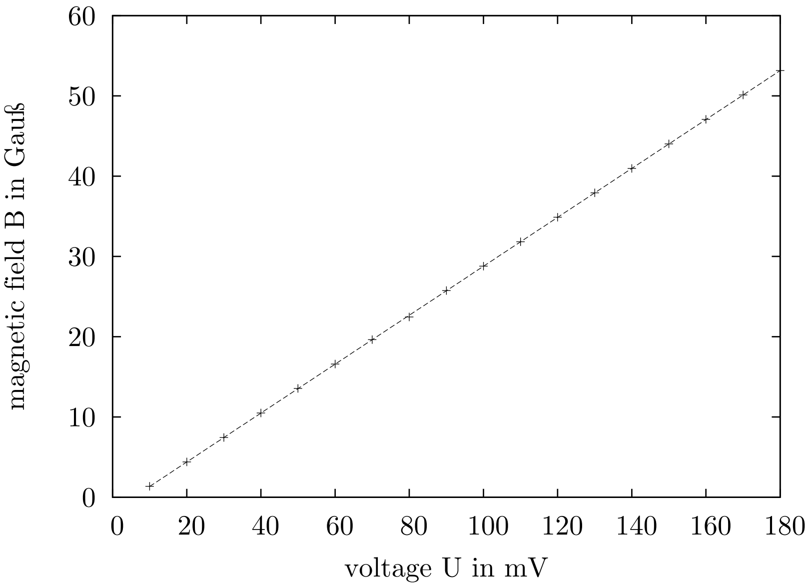
Armin Burgmeier Robert Schittny

1 Determining the magnetic field

For calculating the Landé factor of the muon, we need the magnetic field applied to the second scintillator. For this purpose we obtained the following table, associating the (known) voltage U and the magnetic field B:

U in mV	B in Gauß
10	1,36
20	4,41
30	7,45
40	10,5
50	13,55
60	16,59
70	19,64
80	22,46
90	25,74
100	28,78
110	31,83
120	34,88
130	37,92
140	40,97
150	44,02
160	47,07
170	50,11
180	53,16

We plot these figures and use them for a linear regression:



GnuPlot directly gives us the regression parameters for the equation

$$B = b \cdot U + B_0 \ , \tag{1}$$

yielding (knowing that 1 Gauß equals 0,1 mT)

$$b = (0{,}03048 \pm 0{,}00003)\text{mT/mV} = 0{,}03048\,\text{mT/mV} \pm 0{,}08\% \tag{2}$$
$$B_0 = (-0{,}1707 \pm 0{,}0027)\text{mT} = -0{,}1707\,\text{mT} \pm 1{,}6\% \ . \tag{3}$$

Thus, with a voltage of $U = 123{,}67\,\text{mV}$ we can now calculate our magnetic field:

$$B = 3{,}5988\,\text{mT} \tag{4}$$

Gaussian error propagation gives us

$$\sigma_B = \sqrt{U^2 \cdot \sigma_b^2 + \sigma_{B_0}^2} \ , \tag{5}$$

finally yielding

$$B = 3{,}5988 \pm 0{,}0046\,\text{mT} = 3{,}5988\,\text{mT} \pm 0{,}13\% \ . \tag{6}$$

2 Time calibration of the detector

To be able to calculate the muon lifetime later, we first need to do a time calibration of our detector channels. Therefore, we manually set time delays from zero to 15 ms on a defined signal, obtaining the following values:

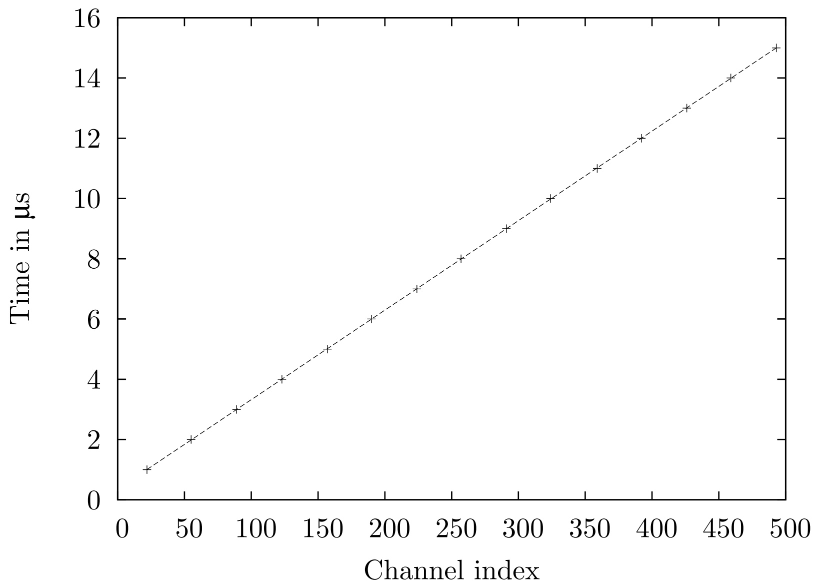


Figure 1: Time calibration of the detector

Again, GnuPlot supplies us with the regression parameters for the equation

$$t(K) = a \cdot K + t_0 \ , \tag{7}$$

with the channel number K:

$$a = 0{,}0297 \pm 2{,}07 \cdot 10^{-5}\ \mu\text{s} = 0{,}0297\ \mu\text{s} \pm 0{,}07\% \tag{8}$$

$$t_0 = 0{,}352 \pm 0{,}006\ \mu\text{s} = 0{,}352\ \mu\text{s} \pm 1{,}74\% \tag{9}$$

3 Lifetime of the muon

The experiment was running since the beginning of the semester, which is about two months. The following plot shows the time between the detection of a muon in the detector and the detection of its decay products on the X axis, and the number of the events on the Y axis. We cut off the first 15 channels (very fast decays), as different effects showed up there (such as a peak at channel 5) which are not related to the lifetime of the muon.

We performed a non-linear fit of the form

$$N\left(K\right) = A\mathrm{e}^{-\frac{K}{\kappa}} + c \tag{10}$$

with the fit parameters A, κ and c since this describes the decay law with a constant underground c assumed.

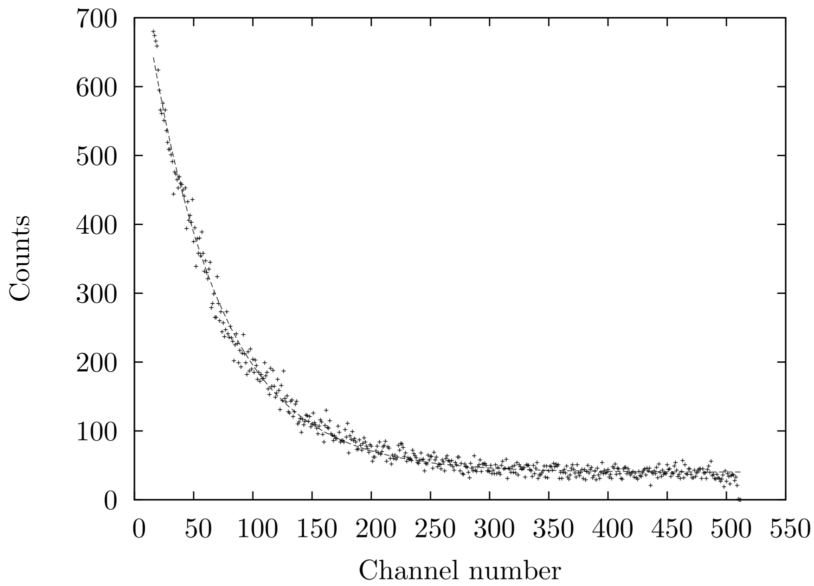


Figure 2: Muon lifetime

The resultant parameter values are

$$A = 779{,}651 \pm 5{,}236 \quad (11)$$
$$\kappa = 62{,}0198 \pm 0{,}5541 \quad (12)$$
$$c = 40{,}0581 \pm 0{,}7823 \quad (13)$$

To gain the muon lifetime from κ, we need to apply the time calbration we did before. For this purpose, we compare our fitted curve with the decay law $\mathrm{e}^{-\frac{t}{\tau}}$:

$$A\mathrm{e}^{-\frac{K}{\kappa}} \overset{!}{=} A_0\mathrm{e}^{-\frac{t(K)}{\tau}} = A_0\mathrm{e}^{-\frac{aK+t_0}{\tau}} = A_1\mathrm{e}^{-\frac{aK}{\tau}} \quad (14)$$

We can thus identify

$$A = A_1 = A_0\mathrm{e}^{-\frac{t_0}{\tau}} \quad (15)$$
$$\frac{K}{\kappa} = \frac{aK}{\tau} \Rightarrow \tau = a\kappa \quad (16)$$

Furthermore, we treat any errors in calibration as systematic ones here, because they would not change if we did multiple measurements. The error of κ (resulting from the fit) becomes the statistical error of the lifetime. We find

$$\begin{aligned}
\tau &= t(\kappa) = a\kappa = 0{,}0297\ \mu\text{s}\cdot\kappa = 1{,}842\ \mu\text{s} && (17)\\
\sigma_{\tau,stat} &= \left|\frac{\partial t}{\partial K}\Delta\kappa\right| = |a\Delta\kappa| = 0{,}0297\ \mu\text{s}\cdot\Delta\kappa = 0{,}017\ \mu\text{s} && (18)\\
\sigma_{\tau,sys} &= \left|\frac{\partial t}{\partial a}\Delta a\right| = |\kappa\Delta a| = \left|\kappa\cdot 2{,}07\cdot 10^{-5}\ \mu\text{s}\right| = 0{,}002\mu\text{s} && (19)
\end{aligned}$$

We can thus give the resultant muon lifetime as

$$\tau = (1{,}842 \pm 0{,}017 \pm 0{,}002)\ \ \mu\text{s} \tag{20}$$

Comparing to the literature value of 2,197 µs we find that our value is somewhat too small. We guess this comes from the same effect because of which we cut off the first 15 channels. If we would cut off even more channels, such as the first 100 ones, then we would find $\tau = 2{,}08$ µs and a higher error, which comes close to the literature value. However, as we don't know what causes this effect we can't handle it properly, and we don't have a good estimation at which channel to do the cut-off.

4 Landé factor of the muon

To find the Landé factor of the muon, we need to find the oscillations due to the magnetic field in the data points. First, we extracted the pure lifetime part of the data by subtracting the underground factor c and dividing by the exponential function determined in the section above. The remainder now should simply show sine-like oscillations.

Therefore, we fitted a curve of the form

$$f(K) = B\cdot\sin(wK+\delta)+d \tag{21}$$

to the data set. Depending on the chosen starting values for the fit parameters B, w, δ and d, we got quite different results, especially for w from which we will find ω later. There were only a few w values to which the fit converged, depending on the starting parameter values. We chose the starting values so that the resulting w matched the data set best when merely looking at the resultant curve.

Another thing to note is that due to low counts in higher channels (especially when subtracting the underground) the fluctuations in those are so high that the values are rather meaningless. We therefore cut off all counts above channel number 256 for the fit.

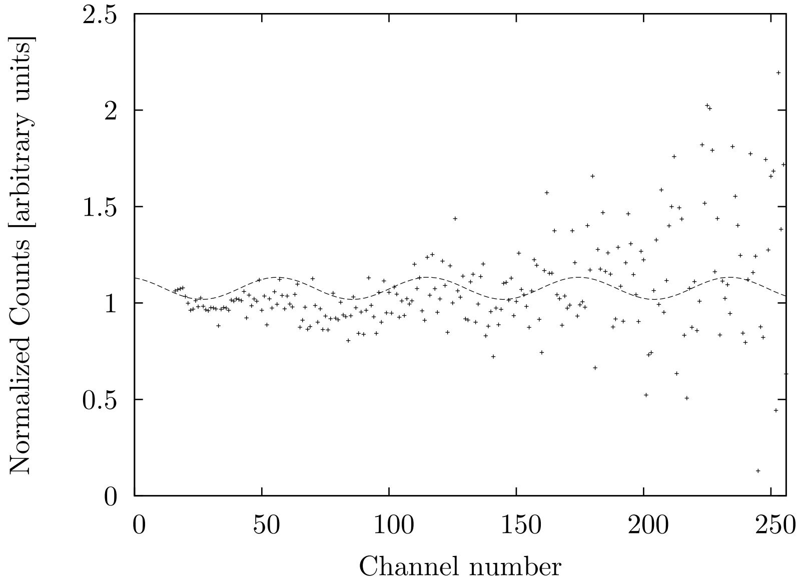


Figure 3: Oscillations due to the magnetic field

Even though the fluctuations are still incrementing with higher channels, we can see that the fitted curve matches the data set in the sense that at the curve's maxima the data set values are greater than in the immediate surroundings, and vice versa for the minima. The fit gives us

$$w = 0{,}106081 \pm 0{,}005871 \tag{22}$$

Again, comparison with the expected curve gives us the relation between w and ω:

$$\sin\left(wK + \delta\right) \stackrel{!}{=} \sin\left(\omega t\left(K\right) + \delta_0\right) = \sin\left(\omega\left(aK + t_0\right) + \delta_0\right) = \sin\left(\omega aK + \delta_1\right) \tag{23}$$

By identification of the constant and K-depending terms, respectively, we find:

$$\delta = \delta_1 = \delta_0 + \omega t_0 \tag{24}$$

$$wK = \omega aK \Rightarrow \omega = \frac{w}{a} \tag{25}$$

Knowing ω, we can determine the Landé factor g, which is given by (according to our preparation):

$$
\begin{aligned}
g &= \frac{\hbar\omega}{\mu_B B} = \frac{2m_\mu \hbar w}{ae\hbar B} = 2{,}337 && (26) \\
\sigma_{g,stat} &= \left|\frac{\partial g}{\partial w}\Delta w\right| = \frac{2m_\mu \Delta w}{aeB} = 0{,}130 && (27) \\
\sigma_{g,sys} &= \sqrt{\left(\frac{\partial g}{\partial a}\Delta a\right)^2 + \left(\frac{\partial g}{\partial dB}\Delta B\right)^2} && (28) \\
&= \sqrt{\left(\frac{2m_\mu w}{a^2 eB}\Delta a\right)^2 + \left(\frac{2m_\mu w}{aeB^2}\Delta B\right)^2} = 0{,}004 && (29)
\end{aligned}
$$

Our final result for the Landé factor is therefore

$$
g = 2{,}337 \pm 0{,}130 \pm 0{,}004 \tag{30}
$$

Again, the literature value of 2,002 is slightly different. We assume this is because of the same reason as the value for τ is different from the literature value since τ has been used to normalize the data set before applying the sine function to it. However, we cannot cut off the lower channels to try to get better results, since then we would only have the higher channels left. Those however have so much fluctuations in the normalized counts that it would be even more difficult to fit a sane sine curve to them.